CALCULATION OF THERMAL DIFFUSION SEPARATION PROCESSES IN A SYSTEM WITH A TRAVELING MAGNETIC FIELD

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The equation of mass transfer in a column is used to calculate the separation factor and the separative power of a device with a traveling magnetic wave.

Separative effects in a gas-discharge device with a traveling magnetic field were first reported in [1]. This showed the possibility of the efficient separation of both gaseous and isotropic mixtures in such systems. In investigating the separation of xenon isotopes in a traveling-wave device it was shown [2,3] that over a certain range of pressures, depending on the system parameters and the plasma properties, radial thermal diffusion plays an important role in the separation process, transforming into a longitudinal effect of separation because of forced "electromagnetic convection." However, estimates of the enrichment factor performed in [3] did not take account of the transverse variation of a number of characteristics of the medium, in particular, the dynamic viscosity of the gas η and the thermal diffusion constant of the mixture α_T. Meanwhile the really important dependence of these parameters on the radial coordinate, which results from the temperature gradient, must lead to a change of both the profile of the circulating flow and the primary enrichment factor, which must be taken into account in determining the total longitudinal concentration gradient.

We calculate below the separation of isotopic mixtures of xenon for the device described in [4], taking account of the radial variation of η and α_T, and estimate the separative power of the device.

As in [3] the investigation is limited to the case of a negligible longitudinal pressure gradient ($\Delta p/p \ll 1$) and plasma parameters which vary slowly along the length of the chamber. In addition, we assume that the medium is only slightly ionized so that thermal diffusion separative processes proceed as in an ordinary neutral gas [5].

In calculating the circulation we use the condition for the equilibrium of the forces acting on the plasma in the axial direction, expressed in terms of quantities averaged over a period of the high-frequency field [3]

$$\frac{\partial p}{\partial z} = \frac{2W}{\pi V_p L R_2} y^2 + \frac{1}{R_2^2} \frac{1}{y} \frac{\partial}{\partial y} y\eta(y) \frac{\partial v}{\partial y}. \tag{1}$$

We solve (1) by taking the dependence of the viscosity on the radial coordinate in the form

$$\eta = \eta_2 [1 + \theta(1 - y^4)], \tag{2}$$

assuming that η varies linearly with the temperature T (according to data in [6] the temperature dependence of the viscosity of Xe is close to linear in the T = 300-950°K range) and that the radial profile T(y) can be approximated by a fourth-degree polynomial

$$T(y) = T_2 [1 + \theta(1 - y^4)]. \tag{3}$$

Integrating (1), using the equation of continuity and the conditions that the velocity is zero at the wall and bounded on the axis, we obtain for the stream function

$$\psi = A \left\{ \frac{\frac{2}{\lambda} - \ln\frac{\lambda+1}{\lambda-1}}{\ln\frac{\lambda^2-1}{\lambda^2}} \left[\lambda \ln\frac{\lambda^2 - y^4}{\lambda^2} + y^2 \ln\frac{(\lambda + y^2)(\lambda - 1)}{(\lambda - y^2)(\lambda + 1)} \right] + y^2 \ln\frac{\lambda^2 - y^4}{\lambda^2 - 1} + \lambda \ln\frac{y^2 + \lambda}{\lambda - y^2} - 2y^2 \right\}. \tag{4}$$

If $\lambda \to \infty$ ($\theta = 0$), which is equivalent to the assumption of constant viscosity of the gas along the radius of the chamber, Eq. (4) takes a form corresponding to the solution in [3].

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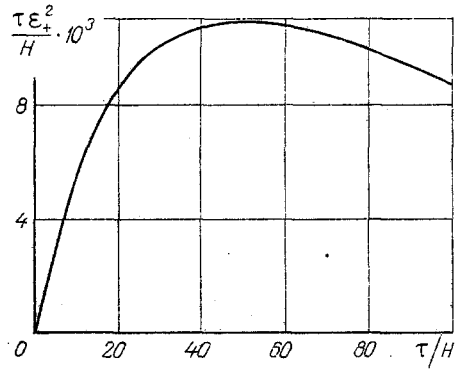

Fig. 1. Dependence of dimensionless quantity $\tau\varepsilon_+^2/H$ on the relative withdrawal τ/H.

The enrichment factor in the longitudinal direction ε_z (for a binary isotopic mixture) is calculated by using the equation of mass transfer in a column derived in [7] by averaging the concentration over the transverse coordinate

$$(K_c + K_d)\frac{dC}{dz} = HC(1-C) + \tau(C - C_\tau). \tag{5}$$

We first integrate (5) for the case of no removal ($\tau = 0$). Using the fact that H, K_c, and K_d are independent of z, we obtain from Eqs. (5), (2), and (3)

$$\varepsilon_z = -\frac{\alpha_T(T_r)\rho W \ln\frac{T_0}{T_2}\int_0^1 \frac{\psi}{A}dy}{12\pi\eta_2^2 V_p\left\{\frac{3+2\theta}{3S_m} + \frac{2S_m\rho^2W^2R_2^2}{24^2\pi^2\eta_2^4V_p^2L^2}\int_0^1 \frac{\left(\frac{\psi}{A}\right)^2 dy}{y[1+\theta(1-y^4)]}\right\}}. \tag{6}$$

We note that in calculating H the stream function was averaged over the transverse coordinate [3], and the temperature dependence of α_T was taken in the form $\alpha_T = B(1 - A/T)$ [7]. We compare the enrichment factor ε_{z0}, calculated from the equations in [3] without taking account of the transverse variation of the medium parameters, with the value of ε_z obtained from (6). Then, if the circulation is relatively weak ($K_c \ll K_d$)

$$\varepsilon_z = -\varepsilon_{z0}\frac{15\alpha_T(T_r)\bar{\eta}^2\int_0^1 \frac{\psi}{A}dr}{4\eta_2^2\left(1+\frac{2}{3}\theta\right)\bar{\alpha}_T}. \tag{7}$$

Taking as the average values $\bar{\eta}$ and $\bar{\alpha}_T$ the viscosity and thermal diffusion constant of xenon at the temperature $\bar{T} = T_2(1 + 4\theta/5)$, we obtain for $T_2 = 300°K$, $T_0 = 850°K$ [8] using the data of [6] ($\eta_2 \approx 2.3\cdot10^{-5}$ kg/m·sec, $\bar{\eta} = 5.2\cdot10^{-5}$ kg/m·sec, $\alpha_T(T_r) \approx 5\cdot10^{-3}$, $\bar{\alpha}_T = 8.7\cdot10^{-3}$)

$$\varepsilon_z = 0.67\varepsilon_{z0}.$$

Thus, the inaccuracy in determining the longitudinal separation factor by using as the average characteristics their values at the temperature $T_2(1 + 4\theta/5)$ is about 30%.

We calculate the absolute value of the enrichment factor of Xe isotopes for the system parameters given in [4,9] for an initial pressure $P_0 = 2\cdot10^{-1}$ torr. Setting $W = 5\cdot10^3$ watts, $R_2 = 0.035$ m, $L = 1$ m, $V_p = 1.3\cdot10^5$ m/sec, $\Delta\mu = 7$ kg/kmole, $S_m = 0.6$, $\alpha_T = 5\cdot10^{-3}$ [6], and $\theta = 1.8$, we obtain for the enrichment factor the value $\varepsilon_z \approx 5.3\%$.

The most important characteristic of the device considered is its separative power [7]. We estimate the value of the separative power in the regime without withdrawal by assuming that the feed with the initial concentration C* enters at the middle of the cross section of the chamber ($z = L/2$), and the enriched product is withdrawn at the end ($z = L$). Then the enrichment factor ε_+ for $C \ll 1$ can be determined from the relation

$$\varepsilon_+ = \frac{C_\tau - C^*}{C^*} = \frac{\exp\left[\dfrac{HL}{2(K_c + K_d)}\left(1 + \dfrac{\tau}{H}\right)\right] - 1}{1 + \dfrac{\tau}{H}\exp\left[\dfrac{HL}{2(K_c + K_d)}\left(1 + \dfrac{\tau}{H}\right)\right]}. \quad (8)$$

Figure 1 shows the dependence of $\tau\varepsilon_+^2$, which characterizes the separative power of the device, on the relative withdrawal τ/H. The maximum value of the separative power is reached at $\tau \simeq 50H \simeq 10^{-8}$ kmole/sec and is $(\tau\varepsilon_+^2)_{max} \simeq 2\cdot10^{-12}$ kmole/sec.

In conclusion we note that the above estimates are valid only for large initial pressures of the mixture, when the relative pressure drop along the length of the system is small, and the principal enrichment mechanism is thermal diffusion.

NOTATION

α_T, thermal diffusion constant; η, dynamic viscosity of the medium; T, temperature; ρ, density; $y = r/R_2$, dimensionless radial coordinate; $P = P_1 + P_2 z/l$, pressure; v, axial component of velocity; V_p, phase velocity of wave; L and R_2, length and inside radius of chamber; w, electric power dissipated in plasma; Δp, longitudinal pressure; drop; z, longitudinal coordinate measured from one end in the direction of wave propagation; η_2, viscosity of the gas at the wall temperature T_2; $\bar{\eta}$, viscosity at the average temperature $\bar{T} = T_2(1 + 4\theta/5)$; θ, constant determining the temperature profile; $T_0 = T_2(1 + \theta)$; $T_\Gamma = T_0T_2/(T_0 - T_2) \times \ln(T_0/T_2)$; $\lambda = \sqrt{(1 + \theta)/\theta}$; C, concentration of heavy component; $\Delta\mu$, difference in molecular weights of isotopes being separated; τ, withdrawal; $S_m = \eta/\rho D$, Schmidt number, $\psi = \int_0^r \rho v r dr$, stream function; $A = \rho W R_2^2/16\pi\eta_2 L\theta^2$; $H = -2\pi\int_0^{R_2}\psi\varepsilon_r dr$; $K_c = 2\pi\int_0^{R_2}\psi^2/\rho D r dr$; $K_d = 2\pi\int_0^{R_2}\rho D r dr$; $\varepsilon_z = \dfrac{C_L/(1-C_L)}{C_0/(1-C_0)} - 1$;

$\varepsilon_+ = (C_\tau - C^*)/C^*$; C_τ and C^*, concentrations in withdrawal and feeding respectively; C_L and C_0, concentrations at $z = L$ and $z = 0$ without withdrawal.

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